Report on the paper
Plant eco-evolution weakens mutualistic interaction with declining pollinator populations
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This paper considers a system of two interacting species, a plant population and a pollinator population, represented by a system of two ordinary differential equations. Dynamics of both populations include intrinsic growth, intraspecific competition as well as mutualistic interactions between both species depending on plant attractiveness.

Authors study evolution of plant attractiveness in an adaptive dynamics way. For that they consider a trade-off between the plant attractiveness and the plant instrinsic growth rate (justified because energy used for attractiveness is not used for intrinsic growth). They illustrate that changing the slope of the trade-off changes the nature of singular strategies of the system: for convexe and linear studied tradeoffs, the singular strategy is a repellor; for concave studied trade-offs, there is a convergence-stable strategy (CSS) and eventually a Garden of Eden. Some analytical calculations are done in Appendix for a particular family of tradeoffs. For convexe tradeoffs, they study the impact of pollinator population decline by studying CSS and Gardens of Eden w.r.t to the intrinsic growth rate r_A of pollinator population. They discuss the evolution of plant attractiveness in case of envionmental degradation for pollinator population (i.e. r_A switches from positive to negative values): attractiveness converges towards 0, leading to pollinator extinction, except if a restoration plan is undertaken sufficiently fast (before a strong descreasing of attractiveness).

In my opinion this paper is interesting. However, I think that the analytical study (in Appendix) for one particular family of tradeoffs is not sufficient to make general conclusions about ESS w.r.t. the shape of the tradeoff as it is done at the end of Section 3. Moreover several misprints are to be corrected in the paper. I give several comments and modifications below that would, in my opinion, help to improve the content of the paper, which can deserve publication after these modifications.

Comments

1. In System (1), N should by A in the first equation;
2. In System (3), α_A should by c_A in the second equation;
3. p.8, l.140: you says that "only concave allocative trade-offs leads to non-invasible strategies.". However, you study only particular trade-offs on the form

$$r_p(\alpha) = \left(1 - \left(\frac{\alpha}{\alpha_{\max}}\right)^s\right)^{1/s}.$$

   I think that it is not sufficient to generalize your results to all trade-off forms. I suggest to modify the sentence into something like "among the particular trade-offs that we study (see Eq. (A2) in Appendix A), only concave allocative trade-offs leads to non-invasible strategies".

4. p.8, l.141 It is not clear why long-term coexistence occurs only for non-invasible strategies. Moreover, from Figure 2, it seems that if α is initialy larger that the repellor, it will converge toward $\alpha_{\max}$ leading to the coexistance of both species.

5. Figure 2. For panels a and b, it seems that $\alpha = 0$ and $\alpha = \alpha_{\max}$ are both CSS.

6. Figures 3 and 4: I think that $\hat{\alpha} = 0$ is a CSS for $r_A < 0$ (as illustrated for example by arrows (4) and (7)).

7. p.13, l.184. Is $s = 2$ a threshold for the existence of Garden of Edens? What is the lanscape for $s = 2$?

8. p.15, l.217 to l.225. I don't think that this result is surprising. In fact evolution of attractiveness is in favor of plant population, whereas degradation of environment affects pollinator population. If pollinator population is not abundant, the benefit of mutualism for plant population is very low. The best strategy for plant population is then to decrease attractiveness in order to increase its instrinsic growth rate.

9. p.16, l.235. Decreasing plant attractiveness decreases pollinator abundance, however does it increases plant abundance? I think that it is the reason why the strategy of the plant evolves toward lower values of attractiveness.

10. the term $\alpha_N \alpha_P - \hat{\epsilon}^2 \gamma_N \gamma_P$ should be $c_P c_A - \hat{\alpha}^2 \gamma_A \gamma_P$ in Eq. (B7);

11. Equation (B11) is derived from (B4) rather than (B6);

12. the term $2c_P\gamma_A r_A$ should be $2c_P\gamma_A r_A \alpha$ (α is missing) in the numerator of (B12);

13. unless I'm mistaken, the derivative of r_P is

$$r'_P(\hat{\alpha}) = r_P(\hat{\alpha}) \frac{1}{\alpha \left(1 - \left(\frac{\hat{\alpha}}{\alpha_{\max}}\right)^{-s}\right)}$$

14. I don't understant how Equation (B14) is derived. However, in order to conclude on the sign of the derivative term, I think that it is better to let $(c_A c_P - \hat{\alpha}\gamma_A\gamma_P)^2$ at the denominator: As $\gamma_A\, r_A - \frac{c_A}{\alpha_{\max}} = -\hat{\alpha}\gamma_A\gamma_P/c_p$, the derivative term is equal to

$$\frac{\gamma_A\gamma_P}{(c_A c_P - \hat{\alpha}\gamma_A\gamma_P)^2} \left[c_A\, c_P + \hat{\alpha}^2\, \gamma_A\gamma_P - 2c_P\hat{\alpha}\frac{\hat{\alpha}\gamma_A\gamma_P}{c_P}\right] = \frac{\gamma_A^2\gamma_P^2}{(c_A c_P - \hat{\alpha}\gamma_A\gamma_P)^2} \left[\underbrace{\frac{c_A c_P}{\gamma_A\gamma_P}}_{=\alpha_{cl}^2} - \hat{\alpha}^2\right]$$

which is positive (when $\hat{\alpha}$ exists) because $\hat{\alpha} \leq \alpha_{\max} < \alpha_{cl}$.

15. Please, go back over all the appendix and correct the layout, for example:
    - equation B12 → Equation (B12) (idem for all equation citations);
    - one bracket is missing in the paragraph between Eq. (B3) and Eq. (B4);
    - you refer to annex A, then name your sections by letters;