

Report on revised version of
**Positive fitness effects help explain the broad range of
Wolbachia prevalences in natural populations**

by P. Karisto, A. Duplouy, C. de Vries, H. Kokko

The authors have adequately revised the manuscript according to my suggestions. Even though the formal analysis of the haplodiploid case is not complete, the numerical results seem to suggest the generality of the presented results. Related to this numerical analysis, I have a question that had escaped me during my first review:

- page 38, line 580: it is not clear to me for which equilibrium (\hat{p}_1 or \hat{p}_2) the condition $0 < \hat{p}_M, \hat{p}_F \leq 1$ applies. I guess, it is meant to apply to both? But in that case, the parameter regime is naturally restricted to $ft < 1$ because otherwise \hat{p}_M (and I believe also \hat{p}_F) of \hat{p}_1 is negative. Hence, the condition $0 < \hat{p}_M, \hat{p}_F \leq 1$ should only apply to \hat{p}_2 to fully explore the stated parameter space. Could the authors clarify what they did? Maybe they could even visualize the locations of \hat{p}_1 and/or \hat{p}_2 ? (based on my small-scale numerical analysis the stability claims in the Appendix seem valid)

Overall, this manuscript is a nice theoretical study that complements the existing theory of *Wolbachia*-induced cytoplasmic incompatibilities. I do not have any remaining objections (besides the one point stated above that requires some clarification).

I have spotted a couple of typos, inconsistencies and inaccuracies (mostly in the appendix) that I mention here:

- page 10, line 215: additional semicolon in the brackets
- inconsistent references to figures: The authors sometimes use Fig., sometimes F/figure and sometimes just state the figure panels (e.g. line 223). I suggest to use one notation and to stay consistent (my preference is Fig. XY).
- page 11, line 223: graph \rightarrow graphs
- legend Figure 2: “ Δp as a function of p ” is not a proper sentence
- Figure 5, panel a: wrong y-axis label
- Figure 7: Maybe this figure can be moved into the appendix. My impression is that the figure does not provide much additional information at the (comparably high) cost of being a completely new figure type compared to the previous figures.
- page 21, line 341: additional dot after ‘unstable’
- page 21, line 348: analysis \rightarrow analyses
- page 24, line 419: eg. \rightarrow e.g.
- Appendix, formulas: Mathematical formulas are typically considered as part of a sentence. Therefore, punctuation rules apply to them. E.g., if an equation is at the end of a sentence a dot should be added (e.g. Eq. (A.2)) or if the sentence continues after the equation, often a comma should close the expression (e.g. Eq. (A.1)). Similarly,

a sequence of equations should be separated by commas (e.g. Eq. (A.1)), but if calculations are done over multiple lines, these should not be separated by commas (e.g. equation after Eq. (B.4)).

- inconsistent reference to equations: Sometimes the authors refer to equations by Eq., sometimes by E/equation and sometimes just by number (with or without braces). Similar to the figures inconsistency, I suggest to use a single format and to stay consistent (my preference is Eq. (XY)).
- page 27, above last formula: $r_F \geq 1/2 \rightarrow r_F \geq 1/2$
- page 27, last line before formula: additional dot at end of sentence
- page 28, sentence after first formula: missing verb(s)
- page 31, line 504: I suggest to add that \hat{p}_1 is below 0 when $ft > 1$. This helped me to understand why (and when) the dynamics change in one dimension.
- page 33, line 529: “ineq. (B6)” – this is wrong reference to an equation. The equation (B6) is an inequality, but one can not refer to it as an ‘inequation’. I suggest to write: ‘left hand side of Eq. (B6)’.
- page 34, line 543: eigenvalues \rightarrow eigenvalue (there is only one dominant eigenvalue)
- page 37, line 572: I \rightarrow we
- page 38, line 577: I suggest to write ‘ $\{0.01 \leq L, t \leq 1\} \times \{0.01 \leq f \leq 3\}$ ’ to avoid confusion about the range of the parameters (I accidentally read 1 times 0.01 and got confused for a bit about the notation). (applies as well to the masculinization section)
- page 38, line 578: delete ‘limits included’ (this is clear from using \leq instead of $<$)
- page 38, line 578/579: I suggest to write 1,000,000 instead of 1 000 000, to avoid the line split at undesired places