Title The inherent multidimensionality of temporal variability: How common and rare species shape stability patterns

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Dear Editors,

Please find below the list of modifications to address the remaining comments.

Thank you again for your investment in our paper.

With best regards,

Jean-François Arnoldi, Michel Loreau and Bart Haegeman

Response to comments of Editors

** There is currently no mention about how the topology of the network may affect your results. This may surprise some readers and it may be worth mentioning explicitly that you did not explore this in this study.*

We have added the following note: "In particular, species interactions are drawn independently between species, leading to relatively simple networks. We do not explore here how more realistic species interaction networks affect our results."

** l. 155: we are unsure that the use of "vs" in this subtitle is justified.*

We think the "vs" is fine; indeed, we want to stress the difference between the perturbation intensity and the community response.

** l. 172, in the main text, σ_{out} is never defined for more than one species, we think it would be relevant to add a link to the relevant equation in the Appendix (B4).*

The variance σ^2_{out} is defined in equation (2). On line 172 we apply this definition to the case of a single species. Equation (B4) is equivalent to equation (2), and hence we did not add a reference to this equation.

** l. 185–186: This sentence is very important and we think it is worth adding the notations defined above, so instead of: "Once intensity is controlled for, perturbations can still differ in how their intensity is distributed and correlated across species," we suggest (if, we are correct): "Once perturbation intensity (σ_{in}) is controlled for, perturbations can still differ in how their intensity (σ_i) is distributed and correlated in time across species ($corr(\xi_i, \xi_j)$)."*

Changed as suggested.

** Caption of figure 2: "(in blue green and red in the rightmost column" a coma is missing; "We derive analytical formulas for the largest value (worst-case scenario) and for the mean value (mean-case scenario)": This sentence can be dropped.*

Changed as suggested.

** In the text the number of species using for simulation is either 40 or 50, which is the correct one? l. 255 "In Fig. 4 we consider a community of $S = 40$ coexisting species"; in Fig. 4's caption: "We consider a community of $S = 40$"; l. 371: "($S = 40$ in our examples)"; in Appendix F: "$S_{pool} = 50$"; l. 244 "we first generate a pool of 50 species"*

We have clarified this as follows: "The species pool consisted of $S_{\text{pool}} = 50$ species [...] The assembled community had $S = 40$ coexisting species."

** Figures 4 and 5: We suggest to add the symbol of the variability in the y-axis label "Induced variability ($\mathcal{V}$)" so the reader can quickly recall what you are referring to.*

Symbol $\mathcal{V}$ added.

** Figure 6: Same as above, we recommend "Invariability ($\mathcal{I}$)" as y-axis label and it might be worth mentioning that it is a measure of stability in the caption.*

We have added the symbol $\mathcal{I}$, and we have specified that stability is quantified as invariability.

** It might be helpful for the reader to stress out that Figures 4 and 5 correspond to a single community but that the results are general.*

We have added the following sentence: "In Fig. 4 we show the variability patterns for a single randomly assembled community, but the results hold more generally (see below)."

** For Figure 5, you should mention explicitly the quantitative difference between weak and strong interactions.*

We have added the following sentence: "From left to right in Fig. 5, the ratios of inter- to intraspecific interaction strength are 0, 0.02 and 0.1 approximately."

** l. 318–325: This paragraph is quite dense and it is hard to follow, it is crucial to explain to the reader how to read May's work in the light of your conceptual framework. Also it might be worth explaining (or citing a paper dealing with this topic) why when the number of interactions increase the number of rare species increases.*

We have simplified this paragraph as follows: "The different relationships between diversity and stability can be understood in terms of the generic variability-abundance patterns of Figs. 4 and 5 (see Appendix H for details). In the case of immigration-type variability, species contributions to variability are proportional to the inverse of their abundance (first panel of Fig. 4). Hence, the worst-case scenario follows the abundance of the rarest species, which rapidly declines with species richness."

** l. 376: "species ability to buffer exogenous perturbations is inversely proportional to their abundance." This sentence is very helpful and so it might be worth adding it in the abstract. Also, would it be more accurate to write "inversely proportional to the square root of their abundance"?*

In our opinion this sentence is too interpretative for the abstract. Indeed, we interpreted "the species' ability to buffer exogeneous perturbations" as the invariability in the case of immigration-type single-species perturbations. We feel that adding the suggested sentence in the abstract could be misleading as the reader cannot know, at this point, what is precisely meant by "exogeneous perturbations". To answer your question, this invariability is proportional to abundance, and the corresponding variability is inversely proportional to abundance (and not the square root), as illustrated in the leftmost panel of Fig. 4.