Review of "Eco-evolutionary dynamics further weakens mutualistic interaction and coexistence under population decline" submitted to *PCI Ecology* by A. Weinbach et al.

July 15, 2021

One reviewer and myself have read the revision of the paper. The reviewer has nothing important to add (except a few minor comments, questions and precision requests). I appreciate a lot your effort to present your model in a more general and comprehensive version. In particular, I find that the introduction is now very good, and that your results are now particularly well presented. Yet, I have to admit I am a bit disappointed that you decided not to change the presentation of the core of the model and sticked to the pollen-pollinator model. On the one hand, I understand that you wanted to keep a less abstract model which can indeed help the reader to understand what is happening. On the other hand, the generality of the model can be missed by readers. In addition, as suggested in the first round of review, if the goal is to modelize a plant-pollinator system, then there is good chance that the equation you proposed are not correct, or under very stringent assumptions only (see Fishman and Hadany 2010 TPB), or for rare plant-pollinator system (for instance geitonogamy). Anyway, I respect your choice, I simply regret a bit that your model might not be understood and read as general as it is (but I will try to emphasize it in my recommendation). As I find your paper interesting, I would be happy to recommend your paper, pending minor corrections requested by the reviewer, and some other minor comments below from myself.

1 Minor comments

- L31: "as illustrated in [9]": be more specific here, please give a quick idea of the content of this paper. It should be obvious for the reader that this paper is relevant in the current context of the present paper.

- L91: "plant-dependent" (check whether a "-" is necessary or not)

- L92 and throughout the paper: please be consistent about the format of figures referencing. I would suggest using capital "F" throughout the paper, using "Fig." abbreviation, and "(-)" when appropriate.

- L100: I do not think the word "dynamical" is necessary (an equilibrium is necessarily defined within a dynamical system?)

- L102: "implies": is there a proof of this implication? If so, where is it given?

- L122: no dot after "modes"

- L173: Can you explain why you decided to only study concave trade-offs? For biological reasons? Technical and methodological reasons? If biological, then why didn't you consider only $s > 1$ from the beginning? If not biological, what other reasons?

- What conclusions can be drawn from the study of convex functions? Any biological interpretation, prediction about the evolution of mutualistic interactions?

- First and second paragraph of section 4: I found these two paragraphs a bit messy as there are partially redundant information, but their goal is different. I have no definitive suggestion about how you could improve the organization of these two paragraphs, but I think it would be important to give it a try since it is here that the reader is introduced in the results of the model. It is important that the reader is not lost in the beginning.

- L189: Figure 4a I suppose?

- L. 192-193: I guess when you decrease the pollinator growth rate in Fig. 4, you assume that the new starting point is at ecological equilibrium, right? I would suggest to add this point explicitly in the text (it helped me to understand how you did your calculations).

- Fig.4b (linked to the previous comment): I think what would help a lot understanding what you did would be to add an arrow also on the plant Figure since, as far as I understand correctly, the plant population also moves on the left, by exactly the same distance and direction than arrow (3) on Fig. 4a.

- I would strongly suggest to add information about plant density in Fig.5: I imagine Figures on plant density should not vary a lot, but I think it is important for the reader to have a clear idea of what's happening to plants.

- L206: It is not immediately clear how you define the "coexistence domain" and how the reader should see its increase in the figure. I would suggest to help the reader a little more here, in particular by defining what you mean by this "domain" and how it is depicted on the figure. It is important because at first, when looking at the size of the dashed line and the position of the red dot, I concluded exactly the reverse than your text: when s increases, the larger the region where pollinator would go extinct when attempting restoration.

- L223: "selects further decreases": please rephrase (I suspect a typo).

Report on the paper
Eco-evolutionary dynamics further weakens mutualistic interaction and coexistence under population decline
(revision of *Plant eco-evolution weakens mutualistic interaction with declining pollinator populations*)
by Avril Weinbach, Nicolas Loeuille and Rudolf P. Rohr
submitted for publication in PCI Ecology

I have only few minor comments in order to improve the new version of the manuscript. My main comments concern, first, the removal of repetitions in Sections A and B, and second, Section C added in Supplementary Material, which is a mathematical analysis of a particular case. It is not clear why this particular case is treated and I suggest to add some explanations about that.

Comments

1. eq. (6): write $r_P(\alpha)$ instead of r_P;

2. Supp. Mat.: some repetitions occur in Sections A and B. I suggest to merge these 2 sections in order to make them clearer.

3. Supp. Mat., top of p.2: Check the english grammar: I think that "the dynamic is" should be replaced by "the dynamics are". Moreover "zeros" should be "zero".

4. Eq (B8): $\hat{\epsilon}^2 \rightarrow \hat{\alpha}^2$

5. Supp. Mat.: Check all equation references and take into account the change of numbering. For example (old) equation (5) is now equation (7). At the beginning of Section "Conditions for invasibility": the trade-off is not defined in Appendix A, but in eq. (6) in the main article. Eq (C3) does not exist; I think you wanted to refer to (C19).

6. I think that Section C in the current state is not really useful. My suggestion: Either explain clearly in the main text that $s = 2$ appears numerically as a threshold for the existence of 2 singular strategies and notice that the mathematical proof is not easy to derive but that we can however prove it in a particular case. Moreover add a sentence like "We prove in this section that $s = 2$ is a threshold for the existence of a second singular strategy in the case $r_A = 0$ for concave trade-off." as introduction of Section C. Or delete this section.